Midterm test for Kwantumfysica 1 - 2012-2013

Friday 28 September 2012, 14:00 - 15:00

READ THIS FIRST:

- Clearly write your name and study number on each answer sheet that you use.
- On the first answer sheet, write clearly the total number of answer sheets that you turn in.
- Note that this test has 3 questions, it continues on the backside of the paper!
- Start each question (number T1, T2,...) on a new side of an answer sheet.
- The test is open book within limits. You are allowed to use the book by Griffiths OR Liboff, and one A4 sheet with notes, but nothing more than this.
- If you get stuck on some part of a problem for a long time, it may be wise to skip it and try the next part of a problem first. The test is only 1 hour.

Useful formulas and constants:

Electron mass	m_e	$= 9.1 \cdot 10^{-31}$ kg
Electron charge	$-e$	$= -1.6 \cdot 10^{-19}$ C
Planck's constant	h	$= 6.626 \cdot 10^{-34}$ Js $= 4.136 \cdot 10^{-15}$ eVs
Planck's reduced constant	$\hbar$	$= 1.055 \cdot 10^{-34}$ Js $= 6.582 \cdot 10^{-16}$ eVs

Problem T1

For this problem, you must write up your answers in Dirac notation.

Consider a quantum system that contains a charged particle with mass m, that has a time-independent Hamiltonian

$$\hat{H} = \hat{T} + \hat{V},$$

where T a kinetic-energy term and V a potential-energy term. The two energy eigenstates of this system with the lowest energy are defined by

$$\begin{aligned} \hat{H}|\varphi_1\rangle &= E_1|\varphi_1\rangle \\ \hat{H}|\varphi_2\rangle &= E_2|\varphi_2\rangle \end{aligned},$$

where $E_1 < E_2$ the two energy eigenvalues, and $|\varphi_1\rangle$ and $|\varphi_2\rangle$ two orthogonal, normalized energy eigenvectors. Energy eigenstates with higher energy do not play a role. The observable $\hat{A}$ is associated with the electrical dipole moment A of this quantum system. For this system,

$$\langle\varphi_1|\hat{A}|\varphi_1\rangle = 0 \quad , \quad \langle\varphi_2|\hat{A}|\varphi_2\rangle = 0 \quad , \quad \langle\varphi_1|\hat{A}|\varphi_2\rangle = \langle\varphi_2|\hat{A}|\varphi_1\rangle = A_0 \ .$$

Note that the states $|\varphi_1\rangle$ and $|\varphi_2\rangle$ are energy eigenvectors, and that they are ***not*** eigenvectors of $\hat{A}$. At some time defined as $t = 0$, the state of the system is (with all c_n a complex-valued constant)

$$|\Psi_0\rangle = c_1\,|\varphi_1\rangle + c_2\,|\varphi_2\rangle = \sqrt{\tfrac{2}{3}}\,|\varphi_1\rangle + e^{i\varphi}\sqrt{\tfrac{1}{3}}\,|\varphi_2\rangle \ .$$

Here φ (a real number) is the phase of the superposition at $t = 0$.

a) [2 points]
Show that as a function of time $t > 0$, the expectation value for $\langle\hat{A}\rangle$ has oscillations at one frequency only. Determine this frequency and the oscillation amplitude (expressed in the constants that are mentioned above), for the case that the system is in $|\Psi_0\rangle$ at $t = 0$.

b) [1 point]
At a time $t = 10$ ns one measures whether the system is in energy eigenstate $|\varphi_1\rangle$ or $|\varphi_2\rangle$. Calculate the probability for the measurement outcome that it is in state $|\varphi_2\rangle$.

Z.O.Z. for question c) of T1

c) [1 point]
For the case of question b), assume that the measurement apparatus is switched off again at $t = 11$ ns. From that moment on the same Hamiltonian as before the measurement is valid for describing the system. Describe the state of the system for $t > 11$ ns. You can assume that it is an ideal measurement apparatus for doing quantum measurements.

Problem T2

Consider a quantum particle with mass m that can only move in the x-direction. It is at some moment in time in the state

$$\Psi(x) = \begin{cases} A\left(1-(x-a_2)^2\right) & \text{for } a_1 < x < a_3 \\ A\left(1-(x-b_2)^2\right) & \text{for } b_1 < x < b_3 \\ 0 & \text{for all other } x \end{cases} .$$

This is a normalized state. The constants are
$a_1 = -3$ nm, $a_2 = -2$ nm, $a_3 = -1$ nm,
$b_1 = +2$ nm, $b_2 = +3$ nm, $b_3 = +4$ nm,
$A = \sqrt{15/32}$ nm$^{-5/2}$
(and the constant 1 in the equation has in fact the unit nm^2).

a) [1 point]
Determine the expectation value $\langle \hat{x} \rangle$ for this state (**hint:** first make a graph of $\Psi(x)$).

b) [1 point]
When one measures the position of the particle when it is in this state, what is the probability to get a measurement outcome between -0.6 nm and + 0.6 nm?

c) [1 point]
When one measures the position of the particle when it is in this state, what is the probability to get a measurement outcome between +3 nm and + 4 nm?

Problem T3

Consider the following quantum system: a particle with mass m that can only move in the x-direction. It is not a free particle, it experiences a position-dependent potential that is constant in time. This is a system with one degree of freedom and with a stationary Hamiltonian.

a) [1 point]
Assume that the particle moves in a potential $V(x) = B_0 \cos(3x) + K_0 x^2$. Write down the time-independent Schrödinger equation for this case, using a representation where all states and operators are expressed as functions of x. That is, you must write it out in a form that shows each term of the equation, and work out each term as a function of x in as much detail as possible with the information that is given.

b) [2 points]
Consider once more a quantum particle of this type, but now it moves in a different potential $V(x)$, that is also constant in time. Derive for this system the time-independent Schrödinger equation from the time-dependent Schrödinger equation. Use the x-representation.